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Application of the Measurement of Shock Detachment Distance at Low Supersonic Speeds

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Introduction

THERE is currently an emphasis on accurate calibration of transonic and supersonic wind tunnels. This includes a precise definition of the spatial variation of flow properties in the test region as well as the fluctuation or temporal variation in flow properties. The shock detachment distance results given in the previous technical note[1] can be of more than academic interest since they demonstrate a very sensitive and repeatable variation with Mach number. A measurement of the shock detachment distance could be used to assess spatial and temporal flow quality independently of the other pressure and hot-wire techniques currently utilized.

Mach Number Resolution

The precision with which Mach number might be established from the shock detachment distance measurement must be addressed first. The variation of the shock standoff distance with Mach number taken from Fig. 3 of Ref. 1 is essential in establishing the resolution which might be achieved. In those aeroballistic range experiments, the standoff distance could be determined from the shadowgraph pictures to within 0.01 body diameters for a 1.5 in. diam sphere. The Mach number resolution that might then be expected in the wind tunnel at low supersonic speeds for this optimistic distance measurement precision and for a less optimistic precision is given in Fig. 1. Clearly the precision with which spatial Mach number variations could be measured in a transonic facility ($M_\infty < 1.2$) is acceptable and even comparable to that achieved with pressure systems. The absolute standoff distance could also be used to set tunnel Mach number precisely or to compare facilities in a fashion similar to the programs currently using other standard transonic bodies. As Mach number increases, the resolution deteriorates significantly based on these model size and precision assumptions.

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Index categories: Subsonic and Transonic Flow, Supersonic and Hypersonic Flow.

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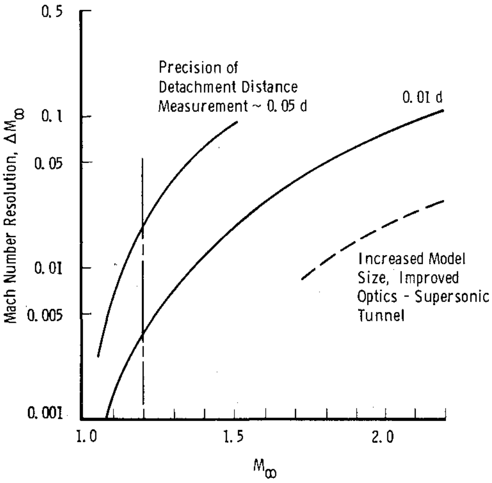

Fig. 1 Subjective analysis of Mach number resolution achievable with shock standoff distance technique.

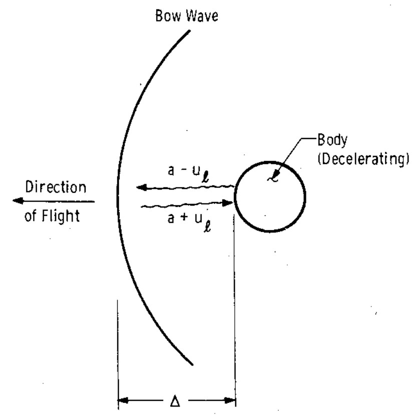

Fig. 2 Schematic of the aerodynamic communication process and resulting response time.

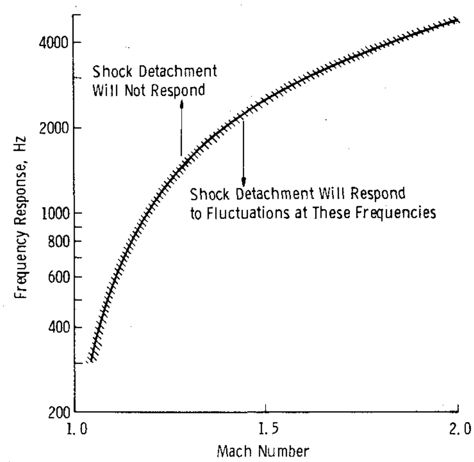

Fig. 3 Probable frequency response of shock detachment distance to flowfield oscillations.

However, model blockage constraints are not as dominant as the Mach number is increased, and the shadowgraph system of most supersonic tunnels is superior. This would permit a significant increase in model size and consideration of the more optimistic precision. Thus the resolution given by the dashed curve in Fig. 1 might be achieved near Mach 2.0.

Mach Number Fluctuations

The most promising application of this technique is the direct measurement of the Mach number fluctuation with time in the wind tunnel. These data could be correlated with the extensive acoustic and hot-wire fluctuation studies currently in progress in transonic facilities. The precision curve given in Fig. 1 would apply for this application with an additional constraint on the fluctuation frequency that could be detected. Fundamental flow response phenomena constrain the frequency domain over which this technique could be applied. To assess the flow response limitations, some insight gained from the tests in the aeroballistic range can be utilized.

Drag forces decelerate the body during an aeroballistic range flight and the rate of deceleration is dependent on the body mass (inertia) as well as the drag force itself. At Mach numbers below 1.01, the shock detachment distance becomes large and does not respond instantaneously to the Mach number decrease of the body if the rate of deceleration is too great. Referring to Fig. 2, the response time can be expressed, in a simplified form, as

$$t_r = \Delta/(a-u_\ell) + \Delta/(a+u_l)$$

where a is the speed of sound and u_ℓ is the average local velocity in the region between the bow wave and the body. The second term is small compared to the first term for low supersonic speeds, so the expression can be simplified to:

$$t_r \simeq \frac{\Delta}{a_\infty(1-M_\ell)}$$

where a_∞ is the freestream speed of sound and M_ℓ is the average local Mach number between the bow wave and the body. This expression describes the flow response phenomena observed in the aeroballistic range tests with a decelerating body very near Mach 1. If it is assumed that the transient response to a Mach number oscillation in the freestream is similar, and that an oscillation with a period as small as four times the response time, t_r, may be observed, an estimate of the frequency response of this system can be made. The calculation of the frequency response of the shock detachment distance is given in Fig. 3. As the Mach number increases, the frequency response becomes high, primarily because of the very short distance between the shock and the body and a resultant short response time. This is offset, however, by a reduction in the Mach number fluctuation that can be resolved, (see Fig. 1). The spectral density of the Mach number variation could thus be determined for correlation with the more conventional fluctuation measurements in the domain of validity. An independent measurement of the Mach number fluctuation in addition to the measurement of other fluctuating aerodynamic properties would be most useful even if the precision of standoff distance measurement limited the comparison to a very low supersonic Mach number (say $M_\infty \sim 1.05$ to 1.1) and the lower portion of the frequency spectrum which exists in modern transonic wind tunnels (frequency < 1000 Hz).

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Natural Frequencies of a Cantilever with Slender Tip Mass

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THE exact frequency equation for a uniform cantilever beam carrying a tip mass has been considered by Pipes,[1] Prescott,[2] Temple and Bickley,[3] and Durvasula.[4] In some situations, the tip mass may be slender in the axial direction, and the center of gravity of the tip mass and its point of attachment to the beam may not coincide. These problems may arise, for instance, in wind-tunnel stings carrying an airplane or a missile model, in large aspect ratio wings carrying heavy tip tanks, in Stockbridge dampers used for damping out galloping of transmission lines, or in launch vehicles with payloads at the tip. In the present Note, the exact frequency equation for such a case is solved with a digital computer, and the results are compared with a perturbation solution given by Bhat and Wagner.[5]

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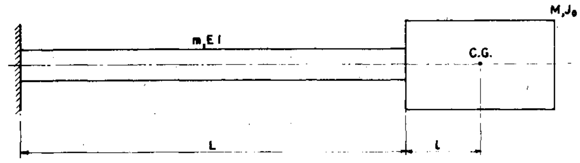

Fig. 1 Schematic of a cantilever with a tip mass.

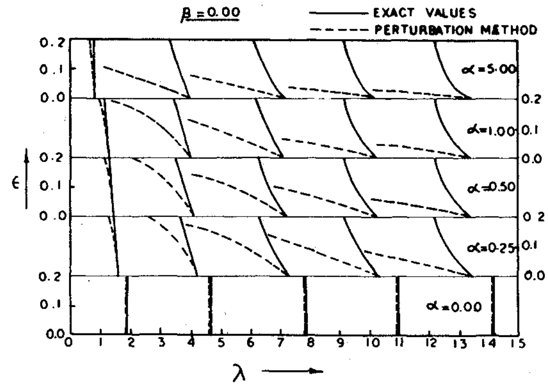

Fig. 2 Comparison of natural frequencies by perturbation method with exact values, $\beta = 0.00$.

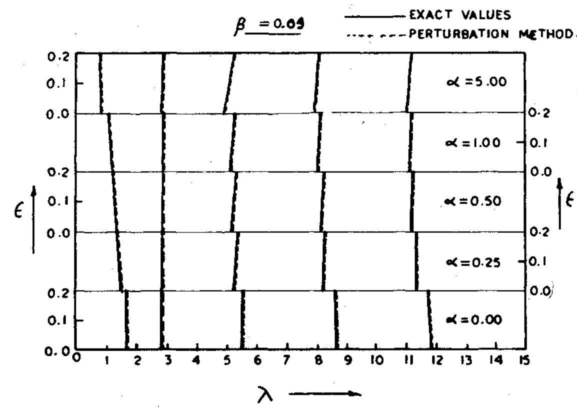

Fig. 3 Comparison of natural frequencies by perturbation method with exact values, $\beta = 0.05$.

A schematic sketch of the uniform beam, carrying slenver tip mass, is shown in Fig. 1. The equation of motion for small deflections is

$$EIy''''(\kappa,t) + m\ddot{y}(\kappa,t) = 0 \tag{1}$$

where primes and dots denote differentiation with regard to space and time, respectively. The boundary conditions at the tip are obtained by variational methods as

$$EIy''(L,t) = -(J_0 + M\ell^2)\ddot{y}'(L,t) - M\ell\ddot{y}(L,t) \tag{2}$$

$$EIy'''(L,t) = M\ddot{y}(L,t)M\ell\ddot{y}'(L,t) \tag{3}$$